# Using Truth Tables to Determine the Validity of Arguments

Truth tables can be used to determine whether a deductive argument is valid or invalid. To do so, follow these steps:

1. Translate the argument you are interested in into symbolic logic, using capital letters to represent each simple proposition, and the logical operators to represent the logical relationship between them.
2. Put a slash (“/”) between each premise, and a double slash (“//”) between the premises and the conclusion.
3. Fill out the truth table in the same way you did before:
   1. Figure out how many rows the truth table needs, and assign truth values to each of the simple propositions.
   2. Follow the order of operations, and determine the truth value of every logical operator in the argument.
   3. Every statement in the argument (both the premises and the conclusion) will have a main operator. You should fill this out last.
4. The argument is **invalid** if there is *any* row in which (a) ALL of the premises are true, but (b) the conclusion is false. (Remember, the definition of validity says that this is *impossible* in a valid argument). If there are NO SUCH ROWS, the argument is **valid.**

Remember that truth tables cannot tell you (a) whether an inductive argument is strong or weak or (b) whether the premises of a valid argument are actually true or not (so, you can’t determine whether the argument is *sound.*)

## Example 1: An Invalid Argument

Here is an example of invalid argument: “If Fido is a dog, then Fido is a mammal. Fido is a mammal. Therefore, Fido is a dog.” Let’s see how this works:

1. We will let D = “Fido is a dog” and M = “Fido is a mammal.”
2. The argument can be represented as follows: / M // D
3. Here is the truth table. I’ve bolded the three main operators.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | / |  | // |  |
| T | **T** | T |  | **T** |  | **T** |
| T | **F** | F | **F** | **T** |
| F | **T** | T | **T** | **F** |
| F | **T** | F | **F** | **F** |

1. We now look to see whether there is any line in which BOTH premises are true, and the conclusion is false. It turns out that there is (the third row of the truth table).

## Example 2: A Valid Argument

Here is an example of a valid argument: “If Fido is a dog, then Fido is a mammal. Fido is NOT a mammal. Therefore, Fido is NOT a dog.” Let’s see how this works:

1. We will let D = “Fido is a dog” and M = “Fido is a mammal.”
2. The argument can be represented as follows:
3. Here is the truth table. I’ve bolded the three main operators.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | / | ~ |  | // | ~ |  |
| T | **T** | T |  | **F** | T |  | **F** | T |
| T | **F** | F | **T** | F | **F** | T |
| F | **T** | T | **F** | T | **T** | F |
| F | **T** | F | **T** | F | **T** | F |

1. We now look to see whether there is any row in which BOTH premises are true, and the conclusion is false. It turns out that there is NO SUCH ROW. This means the argument is valid.

## Example 3: Contradictory premises

One case that people often find weird is the case of contradictory premises. Here’s an example: “If Fido is a dog, then Fido is a mammal. Fido is a dog and Fido is not a mammal. Therefore, Fido is NOT a dog.” (This argument should strike you as a bit weird, right?) Let’s see how this works:

1. We will let D = “Fido is a dog” and M = “Fido is a mammal.”
2. The argument can be represented as follows:
3. Here is the truth table. I’ve bolded the three main operators.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | / | D |  | ~ |  | // | ~ |  |
| T | **T** | T |  | T | **F** | F | T |  | **F** | T |
| T | **F** | F | T | **T** | T | F | **F** | T |
| F | **T** | T | F | **F** | F | T | **T** | F |
| F | **T** | F | F | **F** | T | F | **T** | F |

1. We now look to see whether there is any row in which BOTH premises are true, and the conclusion is false. It turns out that there is no such row, because **it is impossible for the premises to all be true at the same time**. **This means the argument is valid** (it isn’t sound, however, since the premises can’t all be true).

## Corresponding Conditionals: Valid Arguments and Conditional Statements

Every argument has what is called a **corresponding conditional.** This is a conditional statement consisting of (a) an antecedent which conjoins all of the argument’s premises and (b) a consequent consisting of the argument’s conclusion. If the argument is *valid,* then the corresponding conditional will be a *tautology* (if you made a truth table for it, you would discover that the main operator is true on every row). Here are the corresponding conditionals for the earlier arguments:

* Example 1: . This is NOT a tautology, since the argument is invalid.
* Example 2: . This IS a tautology, since the argument is valid.
* Example 3: . This is IS a tautology, since the argument is valid. See the truth table below. The main operator of the corresponding conditional is the ⊃ that separates the antecedent from the consequent.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |
| T | T | T | F | T | F | F | T | **T** | F | T |
| T | F | F | F | T | T | T | F | **T** | F | T |
| F | T | T | F | F | F | F | T | **T** | T | F |
| F | T | F | F | F | F | T | F | **T** | T | F |

## Solved Problems

Problem 1: Translate the following arguments into symbolic form. Then, use truth tables to prove their validity or invalidity. (Again, the truth table software used doesn’t always match the exact symbols we’ve been using).

|  |  |  |  |
| --- | --- | --- | --- |
| Argument | Symbolized | Truth Table | Valid? |
| Either Kurt Vonnegut wrote *Slaughterhouse Five* or Margaret Atwood did. Since Atwood did not write it, Vonnegut must have. | V v A / ~A // V |  | Valid. There is NOT any single line where both premises are true AND the conclusion is false. For example, while the conclusion is false on line 2 (and 4), at least one of the premises is false on these lines as well. |
| Dresden was bombed by the Americans or British. It was bombed by the British. So, it wasn’t bombed by the Americans. | A v B / B // ~A |  | Not valid. The circled row shows that BOTH premises are true AND that the conclusion is false. Since this is possible, we know the argument cannot be valid. |
| Billy was both an optometrist and a time traveler. If Billy traveled through time, he met aliens. So, Billy either met aliens or he was not an optometrist after all. | O ∙ T / T ⊃ A // A v ~O |  | This is valid. Again, there is no line where all of the premises are true and the conclusion is false. |

Problem 2: Produce the CORRESPONDING CONDITIONALS for the arguments above, and evaluate them.

|  |  |  |  |
| --- | --- | --- | --- |
| Argument in Symbolic Form | Corresponding Conditional | Truth Table | Judgement |
| V v A / ~A // V | [(V v A) ∙ ~A] ⊃ V |  | Since this is as a valid argument form (see above), it should be no surprise that the corresponding conditional is a tautology. |
| A v B / B // ~A | ((A v B) & B) > ~A |  | Since this is an invalid argument form, the corresponding conditional is NOT a tautology (and is in fact contingent). |

## Review Questions

1. Use truth tables to prove the validity or invalidity of the following arguments.
   1. Either Bart broke the window or Lisa broke the window. However, Lisa did not break the window. We can conclude that Bart did it.
   2. If Marge went to work, then Maggie stayed home. If Maggie stayed home, Bart snuck out of the house. Bart did sneak out of the house. So, Marge must have went to work.
   3. Bart snuck out of the house if and only if Lisa did her homework. Homer drank beer. So, Bart snuck out of the house.
      1. There are 3 simple positions in this example, so the truth table will require 8 lines.
2. Determine the corresponding conditionals of the arguments in the previous question. Which are tautologies? Which are not?
3. In your own words, explain why any argument with contradictory premises is valid. Can these arguments ever be sound?